

# FYS-6306 QUANTUM THEORY OF MOLECULES AND NANOSTRUCTURES

**Credit units:** 6 ECTS

**Lectures:** 48 h  
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 --> FYS6306 . . .

**Exercises:** 12 x 2 h  
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**Text book:** P.W. Atkins and R.S. Friedman:  
*Molecular Quantum Mechanics*, Chapters 1 – 9  
 (Fifth edition, OXFORD University Press)

**Prerequisites:** Basics of physics and chemistry, quantum mechanics helps

**Examination:** (Thu) Dec 13 2018, (Thu) Jan 31 2019, (Thu) Mar 14 2018

## SCHEDULE 2017

QTMN, 2018

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	WEEK	Lecture	Exercise	Note!
August	35	Tue 1 – 2		
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		Wed 11 – 12	Thu 2	
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		Wed	Thu 6	
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October	43	Tue 25 – 26		
		Wed 27 – 28	Thu	
October	44	Tue 29 – 30		
		Wed 31 – 32	Thu 7	
November	45	Tue 33 – 34		
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