In L-S coupling the spins of all the particles and the orbital angular momenta of all the particles add to yield total **S** and total **L**, which then add to yield **J**.

22. This dependence, which occurs only for (n, γ) reactions with relatively low-energy neutrons, was first measured by Emilio Segré in 1935.

23. The first such resonance was observed unexpectedly in the results of a neutron irradiation of silver conducted by Edoardo Amaldi and others on the morning of October 22, 1934. By 3:00 p.m. that day, Enrico Fermi had developed the correct explanation of the strange phenomenon. The paper describing the discovery was written that evening and delivered to the scientific journal *Ricerca Scientifica* the next morning, less than 24 hours after the discovery!

24. Otto Hahn (1879–1968), German physical chemist, and Fritz Strassmann (1902–1980), German chemist. Hahn recognized that uranium nuclei bombarded with neutrons were breaking apart but carefully avoided characterizing the event as fission since no such thing had been recorded before. He received the 1944 Nobel Prize in Chemistry for the discovery.

25. Actually, Fermi's reactor was the first *constructed* fission reactor. About 2 billion years ago several deposits of natural uranium near what is now Oklo, Gabon (west-central Africa) began chain reactions that continued intermittently for several hundred thousand years at an average power of 100 kW before naturally shutting themselves off. The evidence that verified the discovery of the first of these (in 1972), a fascinating example of scientific detective work, may be found in G. A. Cowan, "A Natural Fission Reactor," *Scientific American*, July 1976. Some of the sites are currently being mined, and efforts to preserve one of the natural reactors as an international historic site are currently under way.

26. An elementary discussion of a magnetic bottle can be found in Section 26-2 in P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, 6th ed., W. H. Freeman and Co., New York, 2008.

27. Godfrey Hounsfield (1919–2004), English engineer, and Allan Cormack (1924–1998), American physicist. They shared the 1979 Nobel Prize in Medicine for the invention of the CT scanner.

28. The radiocarbon dating technique was developed by Willard F. Libby (1908–1980), an American chemist. He received the 1960 Nobel Prize in Chemistry for his work.

Problems

LEVEL I

Section 11-1 The Composition of the Nucleus

11-1. What are the number of protons and the number of neutrons in each of the following isotopes? ^{18}F, ^{25}Na, ^{51}V, ^{84}Kr, ^{120}Te, ^{148}Dy, ^{175}W, and ^{222}Rn.

11-2. Electrons emitted in β decay have energies of the order of 1 MeV or smaller. Use this fact and the uncertainty principle to show that electrons cannot exist inside the nucleus.

11-3. The spin of the ground state of ^{6}Li, which constitutes 7.5 percent of natural lithium, is zero. Show that this value is not compatible with a model of the nucleus that consists of protons and electrons.

11-4. The magnetic moment of ^{14}N is 0.4035 μ_N. Show that this value is not compatible with a model of the nucleus that consists of protons and electrons.

11-5. Suppose that the deuteron really did consist of two protons and one electron. (It doesn't!) Compute the spin and magnetic moment of such a deuteron's ground state and compare the results with the values in Table 11-1.

Section 11-2 Ground-State Properties of the Nuclei

11-6. Give the symbols for at least two isotopes and two isotones of each of the following nuclides: (*a*) ^{18}F, (*b*) ^{208}Pb, and (*c*) ^{120}Sn.

11-7. Give the symbols for at least two isobars and one isotope of each of the following nuclides: (*a*) ^{14}O, (*b*) ^{63}Ni, and (*c*) ^{236}Np.

11-8. Approximating the mass of a nucleus with mass number A as $A \times$ u and using Equation 11-3, compute the nuclear density in SI units.

11-9. Use the masses in the table in Appendix A to compute the total binding energy and the binding energy per nucleon of the following nuclides: (*a*) ^{9}Be, (*b*) ^{13}C, and (*c*) ^{57}Fe.

11-10. Use Equation 11-3 to compute the radii of the following nuclei: (*a*) ^{16}O, (*b*) ^{56}Fe, (*c*) ^{197}Au, and (*d*) ^{238}U.

11-11. Find the energy needed to remove a neutron from (*a*) ^{4}He, (*b*) ^{7}Li, and (*c*) ^{14}N.
11-12. Use the Weizsäcker formula to compute the mass of ^{23}Na. Compute the percent difference between the result and the value in the table in Appendix A.
11-13. Compute the "charge distribution radius" from Equation 11-5 and the "nuclear force radius" from Equation 11-7 for the following nuclides: (*a*) ^{16}O, (*b*) ^{63}Cu, and (*c*) ^{208}Pb.
11-14. ^{39}Ca and ^{39}K are a mirror pair, ^{39}Ca decaying into ^{39}K. Use Equations 11-1 and 11-2 to compute the radius of ^{40}Ca.

Section 11-3 Radioactivity

11-15. The counting rate from a radioactive source is 4000 counts per second at time $t = 0$. After 10 s, the counting rate is 1000 counts per second. (*a*) What is the half-life? (*b*) What is the counting rate after 20 s?
11-16. A certain source gives 2000 counts per second at time $t = 0$. Its half-life is 2 min. (*a*) What is the counting rate after 4 min? (*b*) After 6 min? (*c*) After 8 min?
11-17. A sample of a radioactive isotope is found to have an activity of 115.0 Bq immediately after it is pulled from the reactor that formed it. Its activity 2 h 15 min later is measured to be 85.2 Bq. (*a*) Calculate the decay constant and the half-life of the sample. (*b*) How many radioactive nuclei were there in the sample initially?
11-18. The half-life of radium is 1620 years. (*a*) Calculate the number of disintegrations per second of 1 g of radium and show that the disintegration rate is approximately 1 Ci. (*b*) Calculate the approximate energy of the α particle in the decay $^{226}\text{Ra} \longrightarrow {}^{222}\text{Rn} + \alpha$, assuming the energy of recoil of the Rn nucleus is negligible. (Use the mass table of Appendix A.)
11-19. The counting rate from a radioactive source is 8000 counts per second at time $t = 0$. Ten minutes later the rate is 1000 counts per second. (*a*) What is the half-life? (*b*) What is the decay constant? (*c*) What was the counting rate after 1 minute?
11-20. The counting rate from a radioactive source is measured every minute. The resulting number of counts per second are 1000, 820, 673, 552, 453, 371, 305, 250, . . . (*a*) Plot the counting rate versus time and (*b*) use your graph to estimate the half-life. (*c*) What would be the approximate result of the next measurement after the 250 counts per second?
11-21. ^{62}Cu is produced at a constant rate [e.g., by the (γ, *n*) reaction on ^{63}Cu placed in a high-energy x-ray beam] and decays by β^+ decay with a half-life of about 10 min. How long does it take to produce 90 percent of the equilibrium value of ^{62}Cu?
11-22. The decay constant of ^{235}U is 9.8×10^{-10} y^{-1}. (*a*) Compute the half-life. (*b*) How many decays occur each second in a 1.0 μg sample of ^{235}U? (*c*) How many ^{235}U atoms will remain in the 1.0 μg sample after 10^6 years?
11-23. The decay constant of ^{22}Na is 0.266 y^{-1}. (*a*) Compute the half-life. (*b*) What is the activity of a sample containing 1.0 g of ^{22}Na? (*c*) What is the activity of the sample after 3.5 years have passed? (*d*) How many ^{22}Na atoms remain in the sample at the time?

Section 11-4 Alpha, Beta, and Gamma Decay

11-24. The stable isotope of sodium is ^{23}Na. What kind of radioactivity would you expect of (*a*) ^{22}Na and (*b*) ^{24}Na?
11-25. Using Figure 11-16, find the parameters *A* and *B* in Equation 11-30.
11-26. Make a diagram like Figure 11-18 for the $(4n + 1)$ decay chain that begins with ^{237}Np, a nuclide that is no longer present in nature. (Use Appendix A.)
11-27. Show that the α particle emitted in the decay of ^{232}Th carries away 4.01 MeV, or 98 percent, of the total decay energy.
11-28. ^{7}Be decays exclusively by electron capture to ^{7}Li with a half-life of 53.3 d. Would the characteristics of the decay be altered and, if so, how if (*a*) a sample of ^{7}Be were

placed under very high pressure or (*b*) all four electrons were stripped from each ^{7}Be atom in the sample?

11-29. Compute the energy carried by the neutrino in the electron capture decay of ^{67}Ga to the ground state of ^{67}Zn.

11-30. Compute the maximum energy of the β^- particle emitted in the decay of ^{72}Zn.

11-31. In Example 11-13 we saw that ^{233}Np could decay by emitting an α particle. Show that decay by emission of a nucleon of either type is forbidden for this nuclide.

11-32. With the aid of Figures 11-19 and 11-20, list the energies of all of the possible γ rays that may be emitted by ^{223}Ra following the α decay of ^{227}Th.

11-33. ^{8}Be is very unusual among low-*Z* nuclides: it decays by emitting two α particles. Show why ^{8}Be is unstable toward α decay.

11-34. ^{80}Br can undergo all three types of β decay. (*a*) Write down the decay equation in each case. (*b*) Compute the decay energy for each case.

Section 11-5 The Nuclear Force

11-35. Assuming that the average separation between two protons in ^{12}C is equal to the nuclear diameter, compute the Coulomb force of repulsion and the gravitational force of attraction between the protons. If the nuclear potential "seen" by the protons is 50 MeV for separations up to 3 fm, compare the nuclear force to the other two forces.

11-36. Suppose the range of the nuclear force was 5 fm. Compute the mass (in MeV$/c^2$) of an exchange particle that might mediate such a force.

11-37. The repulsive force that results in the "hard core" of the nucleus might be due to the exchange of a particle, just as the strong attractive force is. Compute the mass of such an exchange particle if the range of the repulsive force equals about 0.25 fm, the radius of the core.

Section 11-6 The Shell Model

11-38. The nuclei listed below have filled *j* shells plus or minus one nucleon. (For example, $^{29}_{14}$Si has the $1d_{5/2}$ shell filled for both neutrons and protons plus one neutron in the $2s_{1/2}$ shell.) Use the shell model to predict the orbital and total angular momentum of these nuclei:

$$^{29}_{14}\text{Si} \quad ^{37}_{17}\text{Cl} \quad ^{71}_{31}\text{Ga} \quad ^{59}_{27}\text{Co} \quad ^{73}_{32}\text{Ge} \quad ^{33}_{16}\text{S} \quad ^{87}_{38}\text{Sr}$$

11-39. Use the shell model to predict the nuclear magnetic moments of the isotopes listed in Problem 11-38.

11-40. The atomic spectral lines of ^{14}N exhibit a hyperfine structure indicating that the ground state is split into three closely spaced levels. What must be the spin of the ^{14}N ground state?

11-41. Which of the following nuclei have closed neutron shells? ^{36}S, ^{50}V, ^{50}Ca, ^{53}Mn, ^{61}Ni, ^{82}Ge, ^{88}Sr, ^{93}Ru, ^{94}Ru, ^{131}In, and ^{145}Eu?

11-42. Sketch diagrams like Figure 11-9 for the ground states of ^{3}H, ^{3}He, ^{14}N, ^{14}C, ^{15}N, ^{15}O, and ^{16}O.

11-43. Which of the following nuclei have closed proton shells: ^{3}He, ^{19}F, ^{12}C, ^{40}Ca, ^{50}Ti, ^{56}Fe, ^{60}Ni, ^{60}Cu, ^{90}Zr, ^{124}Sn, ^{166}Yb, and ^{204}Pb?

11-44. (*a*) Use Figure 11-35 to draw a diagram like Figure 11-9 for ^{13}N. (*b*) What value would you predict for the value of *j*? (*c*) What value would you predict for *j* for the first excited state? (*d*) Draw a diagram like Figure 11-9 for the first excited state. (Is there only one possible?)

11-45. Use Figure 11-35 to predict the values of *j* for the ground states of ^{30}Si, ^{37}Cl, ^{55}Co, ^{90}Zr, and ^{107}In.

Section 11-7 Nuclear Reactions

11-46. Using data from Appendix A, find the Q values for the following reactions: (*a*) $^2H + {}^2H \longrightarrow {}^3H + {}^1H + Q$, (*b*) $^3He(d, p)^4He$, and (*c*) $^6Li + n \longrightarrow {}^3H + {}^4He + Q$.

11-47. (*a*) Find the Q value for the reaction $^3H + {}^1H \longrightarrow {}^3He + n + Q$. (*b*) Find the threshold for this reaction if stationary ^1H nuclei are bombarded with ^3H nuclei from an accelerator. (*c*) Find the threshold for this reaction if stationary ^3H nuclei are bombarded with ^1H nuclei from an accelerator.

11-48. What is the compound nucleus for the reaction of deuterons on ^{14}N? What are the possible product nuclei and particles for this reaction?

11-49. Using data from Appendix A, compute the Q value for the reaction (*a*) $^{12}C(\alpha, p)^{15}N$, and (*b*) $^{16}O(p, d)^{17}O$.

11-50. The cross section for the reaction $^{75}As(n, \gamma)^{76}As$ is 4.5 b for thermal neutrons. A sample of natural As in the form of a crystal 1 cm $\times$ 2 cm that is 30 μm thick is exposed to a thermal neutron flux of 0.95×10^{13} neutrons/cm$^2 \cdot$ s. Compute the rate at which this reaction proceeds. (Natural arsenic is 100 percent ^{75}As. Its density is 5.73 gm/cm^3.)

11-51. Write three different reactions that could produce the products (*a*) $n + {}^{23}Na$, (*b*) $p + {}^{14}C$, and (*c*) $d + {}^{31}P$.

11-52. Write down the correct symbol for the particle or nuclide represented by the x in the following reactions: (*a*) $^{14}N(n, p)x$, (*b*) $^{208}Pb(n, x)^{208}Pb$, (*c*) $x(\alpha, p)^{61}Cu$, (*d*) $^9Be(x, n)^{12}C$, (*e*) $^{16}O(d, \alpha)x$, (*f*) $^{162}Dy(\alpha, 6n)x$, (*g*) $x(d, n)^4He$, (*h*) $^{90}Zr(d, x)^{91}Zr$.

Section 11-8 Fission and Fusion

11-53. A few minutes after the Big Bang the first fusion reaction occurred in the early universe. It was $n + p \longrightarrow d + \gamma$. Compute the Q for this reaction.

11-54. Assuming an average energy release of 200 MeV per fission, calculate the number of fissions per second needed for a 500 MW reactor.

11-55. If the reproduction factor of a reactor is $k = 1.1$, find the number of generations needed for the power level to (*a*) double, (*b*) increase by a factor of 10, and (*c*) increase by a factor of 100. Find the time needed in each case if (*d*) there are no delayed neutrons, so the time between generations is 1 ms, and (*e*) there are delayed neutrons that make the average time between generations 100 ms.

11-56. Write down the several reactions possible when ^{235}U captures a thermal neutron and $1n$, $2n$, $3n$, or $4n$ are produced.

11-57. Assuming an average energy release of 17.6 MeV/fusion, calculate the rate at which ^2H must be supplied to a 500 MW fusion reactor.

11-58. From Figure 11-52, the cross section for the capture of 1.0 MeV neutrons by ^{238}U is 0.02 b. A 5 g sample of ^{238}U is exposed to a total flux of 1.0 MeV neutrons of 5.0×10^{11} per m^2. Compute the number of ^{239}U atoms produced.

11-59. Compute the total energy released in the following set of fusion reactions. This is the proton-proton cycle, the primary source of the Sun's energy.

$$^1H + {}^1H \longrightarrow {}^2H + e^+ + \nu_e$$

$$^2H + {}^1H \longrightarrow {}^3He + \gamma$$

$$^3He + {}^3He \longrightarrow {}^4He + 2\,{}^1H + \gamma$$

11-60. A particular nuclear power reactor operates at 1000 MWe (megawatts electric) with an overall efficiency in converting fission energy to electrical energy of 30 percent. What mass of ^{235}U must fission in order for the power plant to operate for (*a*) one day, (*b*) one year? (*c*) If the energy were provided by burning coal instead of ^{235}U, what would be the answers to (*a*) and (*b*)? (Burning coal produces approximately 3.15×10^7 J/kg.)

11-61. (*a*) Assuming that the natural abundance of deuterium given in Appendix A is reflected in the formation of water molecules, compute the energy that would be released if all the deuterons in 1.0 m^3 of water were fused via the reaction $^2H + {}^1H \rightarrow {}^3He + \gamma$. (*b*) Given that the world's 5.9×10^9 people used 3.58×10^{20} J in 1999, how long (in hours) would the result in part (*a*) have lasted a "typical" person?

11-62. Consider the possible fission reaction

$$n + {}^{235}_{92}U \rightarrow {}^{120}_{48}Cd + {}^{112}_{44}Ru + 3n$$

(*a*) Compute the energy released in the reaction. (*b*) Is this reaction likely to occur? Explain.

Section 11-9 Applications

11-63. A bone claimed to be 10,000 years old contains 15 g of carbon. What should the decay rate of ^{14}C be for this bone?

11-64. A sample of animal bone unearthed at an archeological site is found to contain 175 g of carbon, and the decay rate of ^{14}C in the sample is measured to be 8.1 Bq. How old is the bone?

11-65. The $^{87}Rb/^{87}Sr$ ratio for a particular rock is measured to be 36.5. How old is the rock?

11-66. In a PIXE experiment, an element with $A = 80$ forms 0.001 percent by weight of a thin foil whose mass is 0.35 mg/cm^2. The foil is bombarded with a 250 nA proton beam for 15 minutes. The cross section for exciting the *L* shell is 650 b. If the probability that the excited atom will emit an *L* x ray is 0.60 and the overall efficiency of the x-ray detector is 0.0035, how many counts will the detector record during the 15-minute bombardment?

11-67. The naturally occurring $A = 4n$ decay series begins with ^{232}Th and eventually ends on ^{208}Pb (see Figure 11-18). A particular rock is measured to contain 4.11 g of ^{232}Th and 0.88 g of ^{208}Pb. Compute the age of the rock.

11-68. Compute the resonance frequency of free protons in a magnetic field of (*a*) 0.5×10^{-4} T (the approximate strength of Earth's field), (*b*) 0.25 T, and (*c*) 0.5 T.

11-69. A small piece of papyrus is to be ^{14}C-dated using AMS. During a 10-minute run with the system set to record ^{14}C, 1500 ions are counted. With the system set to transmit $^{12}C^{+3}$ ions, the beam current is 12 μA. (*a*) Compute the $^{14}C/^{12}C$ ratio, assuming both isotopes are transmitted with the same efficiency. (*b*) If the entire sample is consumed in 75 minutes, what was the mass of ^{12}C it contained? (Assume a constant consumption rate and an efficiency of 0.015. (*c*) How old is the sample?

11-70. A wooden spear thrower found in the mountains of southeastern Spain had ^{14}C activity of 2.05 disintegrations per minute per gram. How old is it? (The ^{14}C activity of live wood is 15.6 disintegrations per minute per gram.)

LEVEL II

11-71. Using Equation 11-14 and the constants in Table 11-3, find the *Z* for which $dM/dZ = 0$, that is, the minimum of curves like Figure 11-22*a* for (*a*) $A = 27$, (*b*) $A = 65$, and (*c*) $A = 139$. Do these calculations give the correct stable isobars ^{27}Al, ^{65}Cu, and ^{139}La?

11-72. An empirical expression for distance that α particles can travel in air, called the *range*, is $R(\text{cm}) = (0.31)E^{3/2}$ for *E* in MeV and $4 < E < 7$ MeV. (*a*) What is the range in air of a 5 MeV α particle? (*b*) Express this range in g/cm^2, using $\rho = 1.29 \times 10^{-3}$ g/cm^3 for air. (*c*) Assuming the range in g/cm^2 is the same as that of aluminum ($\rho = 2.70$ g/cm^3), find the range in aluminum in cm for a 5 MeV α particle.

11-73. Show that the average electrostatic energy of a proton-proton pair is about $6ke^2/5R$, where *R* is the separation of the pair and $k = 1/4\pi\epsilon_0$.

11-74. A sample of ^{114}Nd has a mass of 0.05394 kg and emits an average of 2.36 α particles per second. Determine the decay constant and the half-life.

11-75. A sample of radioactive material is found initially to have an activity of 115.0 decays/minute. After 4 d 5 h, its activity is measured to be 73.5 decays/minute.

(a) Calculate the half-life of this material. (b) How long (after $t = 0$) will it take for the sample to reach an activity of 10.0 decays/minute? (c) How long after the time in (b) will it take for the activity to reach 2.5 decays/minute?

11-76. The half-life of ^{227}Th is 18.72 days. It decays by α emission to ^{223}Ra, an α emitter whose half-life is 11.43 days. A particular sample contains 10^6 atoms of ^{227}Th and no ^{223}Ra at time $t = 0$. (a) How many atoms of each type will be in the sample at $t = 15$ days? (b) At what time will the number of atoms of each type be equal?

11-77. The Mössbauer effect was discovered using the decay of the 0.12939 MeV second excited state of ^{191}Ir. The lifetime of this isomer is 0.13 ns. (a) Compute the width Γ of this level. (b) Compute the recoil energy of a free ^{191}Ir atom that emits the 0.12939 MeV photon. (c) Resonant (recoilless) absorption occurs when ^{191}Ir is bound into a lattice. If a Doppler shift equal to Γ destroys the resonance absorption, show that the Doppler velocity v necessary is given by

$$v \approx \frac{c\Gamma}{e}$$

11-78. ^{3}He and ^{3}H are a pair of mirror nuclei. Compute the difference in total binding energy between the two nuclides and compare the result to the electrostatic repulsion of the protons in ^{3}He. Let the protons be separated by the radius of the helium nucleus.

11-79. Use the masses in Appendix A to compute the energy necessary to separate a neutron from ^{47}Ca and ^{48}Ca. From those results determine a value for a_5 in the Weizsäcker formula (Equation 11-14) and compare it with the value in Table 11-3.

11-80. The centripetal force of a nucleus with $I \neq 0$ makes it more stable toward α decay. Use Figure 11-1a and a (classical) argument to show why this is the case.

11-81. (a) Calculate the radii of $^{141}_{56}$Ba and $^{92}_{36}$Kr from Equation 11-3. (b) Assume that after the fission of ^{235}U into ^{141}Ba and ^{92}Kr, the two nuclei are momentarily separated by a distance r equal to the sum of the radii found in (a) and calculate the electrostatic potential energy for these two nuclei at this separation. Compare your result with the measured fission energy of 175 MeV.

11-82. Consider a neutron of mass m moving with speed v_L and colliding head-on with a nucleus of mass M. (a) Show that the speed of the center of mass in the lab frame is $V = mv_L/(m + M)$. (b) What is the speed of the nucleus in the center-of-mass frame before the collision? After the collision? (c) What is the speed of the nucleus in the original lab frame after the collision? (d) Show that the energy of the nucleus after the collision is

$$\frac{1}{2}M(2V)^2 = \left[\frac{4mM}{(m + M)^2}\right]\frac{1}{2}mv_L^2$$

and use this to obtain Equation 11-82.

11-83. Suppose that the Van Dyck painting shown in the photographs on page 554 was irradiated with a thermal neutron flux of 10^{12} neutrons/cm$^2 \cdot$s for 2 h. In terms of the numbers of manganese and phosphorus atoms initially present, determine the activity (a) 2 hours and (b) 2 days after the irradiation stopped. The (n, γ) cross section for ^{31}P is 0.180 b and for ^{55}Mn is 13.3 b. (Both isotopes are 100 percent of the naturally occurring elements.)

11-84. The total energy consumed in the United States in 1 y is about 7.0×10^{19} J. How many kilograms of ^{235}U would be needed to provide this amount of energy if we assume that 200 MeV of energy is released by each fissioning uranium nucleus, that 3 percent of the uranium atoms undergo fission, and that all of the energy-conversion mechanisms used are 25 percent efficient?

11-85. The rubidium isotope ^{87}Rb is a β emitter with a half-life of 4.9×10^{10} y that decays into ^{87}Sr. It is used to determine the age of rocks and fossils. Rocks containing the fossils of early animals contain a ratio of ^{87}Sr to ^{87}Rb of 0.010. Assuming that there were no ^{87}Sr present when the rocks were formed, calculate the age of these fossils.

11-86. In 1989, researchers claimed to have achieved fusion in an electrochemical cell at room temperature. They claimed a power output of 4 W from deuterium fusion reactions in the palladium electrode of their apparatus. (*a*) If the two most likely reactions are

$$^2\text{H} + {}^2\text{H} \longrightarrow {}^3\text{He} + n + 3.27\ \text{MeV}$$

and

$$^2\text{H} + {}^2\text{H} \longrightarrow {}^3\text{H} + {}^1\text{H} + 4.03\ \text{MeV}$$

with 50 percent of the reactions going by each branch, how many neutrons per second would we expect to be emitted in the generation of 4 W of power? (*b*) If one-tenth of these neutrons were absorbed by the body of an 80.0 kg worker near the device and if each absorbed neutron carries an average energy of 0.5 MeV with an RBE of 4, to what radiation dose rate in rems per hour would this correspond? (*c*) How long would it take for a person to receive a total dose of 500 rems? (This is the dose that is usually lethal to half of those receiving it.)

11-87. Neutron activation analysis is used to study a small sample of automotive enamel found at the scene of a hit-and-run collision. The sample was exposed to a thermal neutron flux of 3.5×10^{12} neutrons/cm$^2 \cdot$ s for 2.0 minutes. Placed immediately in a gamma-ray detector, it was found to have an activity of 35 Bq due to ^{60}Co and 115 Bq due to ^{51}Ti. Compute the total amount of each metal in the original sample. (The cross section for ^{59}Co is 19 b; that for ^{50}Ti is 0.15 b.)

11-88. A fusion reactor using only deuterium for fuel would have the following two reactions taking place in it:

$$^2\text{H} + {}^2\text{H} \longrightarrow {}^3\text{He} + n + 3.27\ \text{MeV}$$

and

$$^2\text{H} + {}^2\text{H} \longrightarrow {}^3\text{H} + {}^1\text{H} + 4.03\ \text{MeV}$$

The ^{3}H produced in the second reaction reacts immediately with another ^{2}H to produce

$$^3\text{H} + {}^2\text{H} \longrightarrow {}^4\text{He} + n + 17.7\ \text{MeV}$$

The ratio of ^{2}H to ^{1}H atoms in naturally occurring hydrogen is 1.5×10^{-4}. How much energy would be produced from 4 liters of water if all of the ^{2}H nuclei undergo fusion?

11-89. (*a*) Using the Compton scattering result that the maximum change in wavelength is $\Delta\lambda = 2hc/Mc^2$ and the approximation $\Delta E \approx hc\Delta\lambda/\lambda^2$, show that for a photon to lose an amount of energy E_p to a proton, the energy of the photon must be at least $E = [(1/2)Mc^2E_p]^{1/2}$. (*b*) Calculate the photon energy needed to produce a 5.7 MeV proton by Compton scattering. (*c*) Calculate the energy given a ^{14}N nucleus in a head-on collision with a 5.7 MeV neutron. (*d*) Calculate the photon energy needed to give a ^{14}N nucleus this energy by Compton scattering.

11-90. A photon of energy E is incident on a deuteron at rest. In the center-of-mass reference frame, both the photon and the deuteron have momentum p. Prove that the approximation $p \approx E/c$ is good by showing that the deuteron with this momentum has energy much less than E. If the binding energy of the deuteron is 2.22 MeV, what is the threshold energy in the lab for photodisintegration?

LEVEL III

11-91. (*a*) Compute the binding-energy differences between the two nuclides of the mirror pairs (^{7}Li, ^{7}Be), (^{11}B, ^{11}C), and (^{15}N, ^{15}O). (*b*) From each value computed in (*a*), determine a value of the constant a_3 in Equation 11-14. Compare each value and their average with the value given in Table 11-3.

11-92. (*a*) Differentiate the Weizsäcker empirical mass formula with respect to Z, as in Problem 11-46, and show that the minima of the constant A curves that result, that is, Z values for the most stable isotopes, are given by

$$Z = \frac{A}{2}\left[\frac{1 + \dfrac{(m_n - m_p)c^2}{4a_4}}{1 + \dfrac{a_3 A^{2/3}}{4a_4}}\right]$$

(*a*) Determine the atomic number for the most stable nuclides for $A = 29$, 59, 78, 119, and 140. (*c*) Compare the results in (*b*) with the data in Appendix A and discuss any differences.

11-93. (*a*) Use Figure 11-35 to make a diagram like Figure 11-9 for the ground state of ^{11}B. What do you predict for the value of j for this state? (*b*) The first excited state of ^{11}B involves excitation of a proton. Draw the diagram for this state and predict its j value. (*c*) The j value for the second excited state is $5/2$. Draw a diagram of the nucleons like Figure 11-9 that could account for that value. (*d*) Repeat parts (*a*) and (*b*) for ^{17}O, where the excitation of the first excited state involves a neutron. (*e*) The j value for the second excited state of ^{17}O is $1/2$. Draw a diagram like Figure 11-9 that would explain that value.

11-94. Approximately 2000 nuclides remain to be discovered between the proton and neutron driplines in Figure 11-15*b*. Consider those that lie on the energy parabola (see Figure 11-22*a*) for $A = 151$, whose only stable isotope is ^{151}Eu. (*a*) From the data in Appendix A, draw an accurate diagram of the $A = 151$ parabola showing known nuclides *and* those yet to be discovered between $Z = 50$ and $Z = 71$. (*b*) Determine where the edges of the driplines lie for $A = 151$, that is, the lowest mass isotopes for which spontaneous proton or neutron emission becomes possible.

11-95. There are theoretical reasons to expect that a cluster of relatively long-lived nuclides will exist in the neighborhood of the doubly magic nucleus with $Z = 126$ and $N = 184$, the latter being the next magic number beyond 126 predicted by the shell model. (*a*) Compute the mass of this exotic nucleus using Equation 11-14. (*b*) Computing the necessary masses of the nearby nuclei, predict the decay modes that would be available to the doubly magic nucleus.

11-96. Assume that a neutron decays into a proton plus an electron without the emission of a neutrino. The energy shared by the proton and electron is then 0.782 MeV. In the rest frame of the neutron, the total momentum is zero, so the momentum of the proton must be equal and opposite that of the electron. This determines the relative energies of the two particles, but because the electron is relativistic, the exact calculation of these relative energies in somewhat difficult. (*a*) Assume that the kinetic energy of the electron is 0.782 MeV and calculate the momentum p of the electron in units of MeV/c. (*Hint:* Use Equation 2-32.) (*b*) From your result for (*a*), calculate the kinetic energy $p^2/2m_p$ of the proton. (*c*) Since the total energy of the electron plus proton is 0.782 MeV, the calculation in (*b*) gives a correction to the assumption that the energy of the electron is 0.782 MeV. What percentage of 0.782 MeV is this correction?

11-97. Radioactive nuclei with a decay constant of λ are produced in an accelerator at a constant rate R_p. The number of radioactive nuclei N then obeys the equation $dN/dt = R_p - \lambda N$. (*a*) If N is zero at $t = 0$, sketch N versus t for this situation. (*b*) The isotope ^{62}Cu is produced at a rate of 100 per second by placing ordinary copper (^{63}Cu) in a beam of high-energy photons. The reaction is

$$\gamma + {}^{63}\text{Cu} \rightarrow {}^{62}\text{Cu} + n$$

^{62}Cu decays by β decay with a half-life of 10 minutes. After a time long enough so that $dN/dt \approx 0$, how many ^{62}Cu nuclei are there?

11-98. The $(4n + 3)$ decay chain begins with ^{235}U and ends on ^{207}Pb. (*a*) How many α decays are there in the chain? (*b*) How many β decays are there? (*c*) Compute the total energy released when one ^{235}U atom decays through the complete chain. (*d*) Assuming no energy escapes, determine the approximate temperature rise of 1 kg of ^{235}U metal over the period of 1 year.

11-99. Energy is generated in the Sun and other stars by fusion. One of the fusion cycles, the proton-proton cycle, consists of the following reactions:

$$^{1}H + {}^{1}H \longrightarrow {}^{2}H + \beta^{+} + \nu_e$$

$$^{1}H + {}^{2}H \longrightarrow {}^{3}He + \gamma$$

followed by either

$$^{3}He + {}^{3}He \longrightarrow {}^{4}He + {}^{1}H + {}^{1}H$$

or

$$^{1}H + {}^{3}He \longrightarrow {}^{4}He + \beta^{+} + \nu_e$$

(*a*) Show that the net effect of these reactions is

$$4\,{}^{1}H \longrightarrow {}^{4}He + 2\beta^{+} + 2\nu_e + \gamma$$

(*b*) Show that the rest mass energy of 24.7 MeV is released in this cycle, not counting the 2×0.511 MeV released when each positron meets an electron and is annihilated according to $e^{+} + e^{-} \longrightarrow 2\gamma$. (*c*) The Sun radiates energy at the rate of about 4×10^{26} W. Assuming that this is due to the conversion of four protons into helium plus γ rays and neutrinos, which releases 26.7 MeV, what is the rate of proton consumption in the sun? How long will the Sun last if it continues to radiate at its present level? (Assume that protons constitute about half the total mass of the Sun, which is about 2×10^{30} kg.)

11-100. The fusion reaction between ^{2}H and ^{3}H is

$$^{3}H + {}^{2}H \longrightarrow {}^{4}He + n + 17.7\ \text{MeV}$$

Using the conservation of momentum and the given Q value, find the final energies of both the ^{4}He nucleus and the neutron, assuming that the initial momentum of the system is zero.

11-101. (*a*) A particular light-water ^{235}U-fueled reactor had a reproduction factor of 1.005 and an average neutron lifetime of 0.08 s. By what percentage will the rate of energy production by the reactor increase in 5 s? (*b*) By what fraction must the neutron flux in the reactor be reduced in order to reduce the reproduction factor to 1.000?

11-102. Compute the reproduction factor for uranium enriched to (*a*) 5 percent and (*b*) 95 percent in ^{235}U. Compute the corresponding fission rate doubling time in each case. Assuming no loss of neutrons and the release of 200 MeV/fission, at what rate will energy be produced in each case 1.0 s after the first fission occurs?

22. They form nine combinations, just like the mesons, but for the gluons the ninth combination is really a singlet and, hence, is independent.
23. Since no theory of quantum gravity complementing QED and QCD exists, current efforts to develop GUTs include only the strong and electroweak interactions.
24. Raymond Davis, Jr. (1914–2006), American physicist, and John Bahcall (1934–2005) American physicist. His measurements won Davis a share of the 2002 Nobel Prize in Physics.
25. Theories in which the interaction is determined by the invariance of the theory (i.e., its mathematical equations) under particular transformations are called *gauge theories*. For example, classical electrodynamics is a gauge theory (although not usually referred to as such), as are QED and QCD. Historically, interactions were "figured out" by clever physicists on the basis of experimental evidence. A bit of a surprise, Schrödinger's wave mechanics is not a gauge theory.

Problems

LEVEL I

Section 12-1 Basic Concept

12-1. Two pions at rest annihilate according to the reaction $\pi^+ + \pi^- \rightarrow \gamma + \gamma$. (*a*) Why must the energies of the two gamma rays be equal? (*b*) Find the energy of each gamma ray. (*c*) Find the wavelength of each gamma ray.
12-2. Find the minimum energy of the photon needed for the following reactions: (*a*) $\gamma \rightarrow \Lambda^+ + \pi^-$, (*b*) $\gamma \rightarrow p + \bar{p}$, and (*c*) $\gamma \rightarrow \mu^- + \mu^+$.
12-3. Draw two different Feynman diagrams for each of the following events. (*a*) $e^+ + e^- \rightarrow e^+ + e^-$; (*b*) $\gamma + e^- \rightarrow \gamma + e^-$.
12-4. Draw a Feynman diagram illustrating each of the following scattering events: (*a*) electron-electron, (*b*) electron-positron, and (*c*) Compton effect.
12-5. Find (*a*) the energy of the electron, (*b*) the energy of the ^{32}S nucleus, and (*c*) the momentum of each in the decay $^{32}\text{P} \rightarrow {}^{32}\text{S} + e^-$, assuming no neutrino in the final state ($n \rightarrow p + e^-$). (The rest mass of ^{32}P is 31.973762 u.)
12-6. The fate of an antiproton is usually annihilation via the reaction $p + \bar{p} \rightarrow \gamma + \gamma$. Assume that the proton and antiproton annihilate at rest. (*a*) Why must there be two photons rather than just one? (*b*) What is the energy of each photon? (*c*) What is the wavelength of each photon? (*d*) What is the frequency of each photon?
12-7. Figure 12-2 shows the production of the first antiproton. It was produced by the reaction $p + p \rightarrow p + p + p + \bar{p}$ and required a minimum kinetic energy of 5.6 GeV. (The proton beam energy was actually 25 GeV.) Less energy would be required by either of the following reactions. Why is neither of them a possible alternative? Justify your answer. (*a*) $p + p \rightarrow p + e^- + e^+ + \bar{p}$ (*b*) $p + p \rightarrow p + \bar{p}$
12-8. Positronium is a bound state of an electron and a positron (see Section 2-4). Its lifetime expressed in natural units used by particle physicists ($\hbar = c = 1$) is $\tau = 2/m\alpha^5$, where m = electron mass and α = the fine structure constant. Use dimensional analysis (*a*) to include $\hbar$ and c in the expression for τ and (*b*) to compute the value of τ.

Section 12-2 Fundamental Interactions and the Force Carriers

12-9. Name the interaction responsible for each of the following decays:
(*a*) $n \rightarrow p + e^- + \bar{\nu}_e$
(*b*) $\pi^0 \rightarrow \gamma + \gamma$
(*c*) $\Delta^+ \rightarrow \pi^0 + p$
(*d*) $\pi^+ \rightarrow \mu^+ + \nu_\mu$
12-10. Which of the following decays—$\pi^0 \rightarrow \gamma + \gamma$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$—would you expect to have the longer lifetime? Why?
12-11. Of the following reactions, which are allowed to proceed via the weak interaction and which are forbidden? Justify your answer.

(*a*) $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$
(*b*) $p + e^- + \nu_e \rightarrow e^- + \pi^+ + p$
(*c*) $\Lambda^0 \rightarrow \pi^+ + e^- + \overline{\nu_e}$
(*d*) $p + \nu_\mu \rightarrow \mu^+ + n$

12-12. Which of the four fundamental interactions is most likely responsible for the following reactions?
(*a*) ^{16}O (excited state) $\rightarrow$ ^{16}O (ground state) + γ
(*b*) $\nu_e + e \rightarrow \nu_e + e$
(*c*) $p + \overline{p} \rightarrow \gamma + \gamma$
(*d*) $p + \overline{\nu}_e \rightarrow n + e^+$
(*e*) $\pi^0 + p \rightarrow \pi^0 + p$
(*f*) $^3\text{H} \rightarrow {}^3\text{He} + e^- + \overline{\nu}_e$

12-13. Using the information concerning the neutrinos from SN1987A, including Figure 12-33, compute an upper limit to the mass of the electron neutrino.

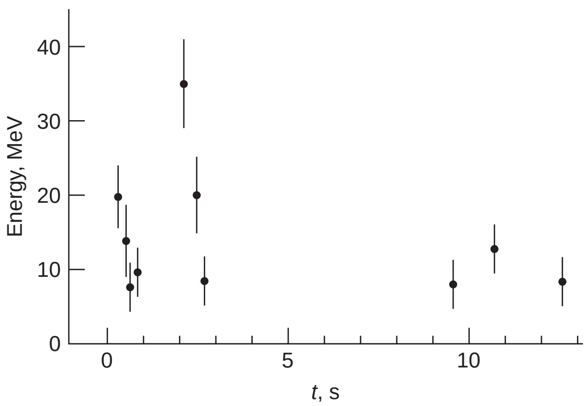

FIGURE 12-33 Electron antineutrino energy versus arrival time in the Kamiokande detector in Japan for antineutrinos emitted by the supernova 1987A. The spread in arrival times (about 13 s) permits a calculation of an upper limit to the mass of the $\overline{\nu}_e$.

12-14. The rest energies of the Σ^+ and Σ^- are slightly different, but those of the π^+ and π^- are exactly the same. Explain this difference in behavior.

12-15. Draw Feynman diagrams of the following decays:
(*a*) $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$
(*b*) $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$
(*c*) $\tau^- \rightarrow \mu^- + \overline{\nu}_\mu + \nu_\tau$

Section 12-3 Conservation Laws and Symmetries

12-16. What is the uncertainty in the rest energies of the following particles? (*a*) $\Lambda(1670)$, (*b*) $\Sigma(2030)$, (*c*) $\Delta(1232)$.

12-17. State which of the decays or reactions that follow violate one or more of the conservation laws, and give the law or laws violated in each case.
(*a*) $p \rightarrow n + e^+ + \overline{\nu}_e$
(*b*) $n \rightarrow p + \pi^-$
(*c*) $e^+ + e^- \rightarrow \gamma$
(*d*) $p + \overline{p} \rightarrow \gamma + \gamma$
(*e*) $\nu_e + p \rightarrow n + e^+$
(*f*) $p \rightarrow \pi^+ + e^+ + e^-$

12-18. The neutral pion decays 99 percent of the time by the reaction $\pi^0 \rightarrow 2\gamma$. The π^- decays by the reaction $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$. (*a*) Assuming the π^0 to consist of a $u\overline{u}$ quark pair, show how the 2γ occurs. (*b*) Why is a π^0 decay to a single photon not possible? (*c*) The π^- is a $\overline{u}d$ quark combination. Draw a Feynman diagram for the π^- decay.

12-19. Determine the change in strangeness in each reaction that follows, and state whether the reaction can proceed via the strong interaction, the electromagnetic interaction, the weak interaction, or not at all:
(*a*) $\Omega^- \rightarrow \Xi^0 + \pi^-$, (*b*) $\Xi^0 \rightarrow p + \pi^- + \pi^0$, and (*c*) $\Lambda^0 \rightarrow p + \pi^-$

12-20. Determine the change in strangeness for each decay, and state whether the decay can proceed via the strong interaction, the electromagnetic interaction, the weak interaction, or not at all:
(*a*) $\Omega^- \rightarrow \Lambda^0 + \bar{\nu}_e + e^-$, (*b*) $\Sigma^+ \rightarrow p + \pi^0$, and (*c*) $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

12-21. The rules for determining the isospin of two or more particles are the same as those for combining angular momentum. For example, since $T = 1/2$ for nucleons, the combination of two nucleons can have either $T = 1$ or $T = 0$ or may be a mixture of these isospin states. Since $T_3 = +1/2$ for the proton, the combination $p + p$ has $T_3 = +1$ and therefore must have $T = 1$. Find T_3 and the possible values of T for the following:
(*a*) $n + n$
(*b*) $n + p$
(*c*) $\pi^+ + p$
(*d*) $\pi^- + n$
(*e*) $\pi^+ + n$

12-22. Which of the following decays are allowed and which are forbidden? If the decay is allowed, state which interaction is responsible. If it is forbidden, state which conservation law its occurrence would violate.
(*a*) $\pi^- \rightarrow e^- + \gamma$
(*b*) $\pi^0 \rightarrow e^- + e^+ + \nu_e + \bar{\nu}_e$
(*c*) $\pi^+ \rightarrow e^- + e^+ + \mu^+ + \nu_\mu$
(*d*) $\Lambda^0 \rightarrow \pi^+ + \pi^-$
(*e*) $n \rightarrow p + e^- + \bar{\nu}_e$

12-23. For each of the following particles, write down two possible decays that satisfy all conservation laws: (*a*) Ω^-, (*b*) Σ^+, (*c*) Λ^0, (*d*) π^0, and (*e*) K^+.

12-24. Consider the following reactions:

$$K^- + p \rightarrow K^0 + K^+ + \Omega^-$$

followed by

$$\Omega^- \rightarrow \Xi^0 + \pi^-$$

Given that $B = 1$ for the proton and $B = 0$ for mesons and that baryon number is conserved, determine the baryon number of the Ω^- and the Ξ^0.

12-25. Which of the following decays and reactions conserve strangeness?
(*a*) $\bar{p} + p \rightarrow \gamma + \gamma$
(*b*) $\Xi^- \rightarrow \pi^- + \Lambda^0$
(*c*) $\Sigma^+ \rightarrow \Lambda^0 + \pi^+$
(*d*) $\pi^- + p \rightarrow \pi^- + \Sigma^+$
(*e*) $\Omega^- \rightarrow \Xi^- + \pi^0$

Section 12-4 The Standard Model

12-26. Find the baryon number, charge, isospin, and strangeness for the following quark combinations and identify the corresponding hadron: (*a*) *uud*, (*b*) *udd*, (*c*) *uuu*, (*d*) *uss*, (*e*) *dss*, (*f*) *suu*, and (*g*) *sdd*.

12-27. Find the baryon number, charge, isospin, and strangeness for the following quark combinations and identify the corresponding hadron (the charge and strangeness of the antiquarks are the negatives of those of the corresponding quarks, as with any other particle-antiparticle pair): (*a*) $u\bar{d}$, (*b*) $\bar{u}d$, (*c*) $u\bar{s}$, (*d*) $s\bar{s}$, and (*e*) $\bar{d}s$.

12-28. Draw two Feynman diagrams that represent the decay of the anti-bottom quark.

12-29. Some quark combinations can exist in two or more isospin states, with each state corresponding to a different hadron. One such combination is *uds*. (*a*) What is the value of T_3 for this combination? (*b*) What are the possible values of total isospin T for this combination? (*c*) Find the baryon number, charge, and strangeness of this combination, and identify the hadron corresponding to each isospin state.

12-30. The Δ^{++} particle is a baryon that decays via the strong interaction. Its strangeness, charm, topness, and bottomness are all zero. What combination of quarks gives a particle with these properties?

12-31. Compute the approximate range of a weak interaction mediated by a W^+.

12-32. One mode of weak decay of the $\overline{K}^0$ is

$$\overline{K}^0 \to \pi^+ + \mu^- + \overline{\nu}_\mu$$

Showing the quark content of the particles, draw the Feynman diagram of this so-called semileptonic decay.

12-33. The Λ^0 undergoes a weak decay as follows: $\Lambda^0 \to p + \pi^-$. Showing the quark content of the particles, draw the Feynman diagram of this so-called nonleptonic decay.

12-34. Show that the neutron cannot undergo the weak decay shown for the Λ^0 in Problem 12-32.

12-35. The decay of the Λ^0 shown in Problem 12-33 can also proceed via the strong interaction. Showing the quark content of the particles, draw the Feynman diagram that illustrates the strong decay of the Λ^0.

12-36. The X^0 (1193) can be produced by the reaction $K^- + p \to \pi^0 + X^0$. (*a*) Determine the baryon, strangeness, charm, and bottom quantum numbers of the X^0 (1193). (*b*) From your answer to (*a*), what is the quark content of the X^0 (1193)?

12-37. Find a possible combination of quarks that gives the correct values for electric charge, baryon number, and strangeness for (*a*) K^+ and (*b*) K^0.

12-38. The D^+ meson has strangeness 0, but it has charm of +1. (*a*) What is a possible quark combination that will give the correct properties for this particle? (*b*) Repeat (*a*) for the D^- meson, which is the antiparticle of the D^+.

12-39. The lifetime of the Σ^0 is 6×10^{-20} s. The lifetime of the Σ^+ is 0.8×10^{-10} s and that of the Σ^- is 1.48×10^{-10} s, nearly twice as long. How can these differences in lifetimes between members of the same isospin multiplet be explained?

Section 12-5 Beyond the Standard Model

12-40. Grand unification theories predict that the proton is unstable. If that turns out to be true, why does it mean that baryon number is not conserved? If leptons and quarks are interchangeable at the unification energy, does this mean that there is a new, conserved "leptoquark number"?

12-41. GUTs predict a lifetime of about 10^{32} y for the proton. If that is the case, how many protons will decay each year in the world's oceans? (Assume the average depth of the oceans to be 1 km and that they cover 75 percent of Earth's surface.)

12-42. Protons might decay via a number of different modes. What conservation laws are violated by the following possibilities?

(*a*) $p \to e^+ + \Lambda^0 + \nu_e$
(*b*) $p \to \pi^+ + \gamma$
(*c*) $p \to \pi^+ + K^0$

LEVEL II

12-43. Find a possible quark combination for the following particles: (*a*) $\overline{n}$, (*b*) Ξ^0, (*c*) Σ^+, (*d*) Ω^-, and (*e*) Ξ^-.

12-44. State the properties of the particles made up of the following quarks: (*a*) *ddd*, (*b*) $u\overline{c}$, (*c*) $u\overline{b}$, and (*d*) $\overline{sss}$.

12-45. Show that the Z^0 cannot decay into two identical zero-spin particles.

12-46. Consider the following decay chain:

$$\begin{aligned}
&\Xi^0 \rightarrow \Lambda^0 + \pi^0 \\
&\Lambda^0 \rightarrow p + \pi^- \\
&\pi^0 \rightarrow \gamma + \gamma \\
&\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
&\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu
\end{aligned}$$

(*a*) Are all the final products shown stable? If not, finish the decay chain. (*b*) Write the overall decay reaction for Ξ^0 to the final products. (*c*) Check the overall decay reaction for the conservation of electric charge, baryon number, lepton number, and strangeness. (*d*) In the first step of the chain, could the Λ^0 have been a Σ^0?

12-47. There are six hadrons with quantum numbers (Q,U,S,C,B) = (2,1,0,1,0); (0,1,−2,1,0); (0,0,1,0,−1); (0,−1,1,0,0); (0,1,−1,1,0); (−1,1,−3,0,0). Determine the quark content of each hadron.

12-48. Two neutrinos of different energies E_1 and E_2 emitted by supernova SN1987A arrive at Earth at different times. Let $E_1 = 20$ MeV and $E_2 = 5$ MeV and assume that the mass of the neutrino is $2.2\ \text{eV}/c^2$. Because their total energy is much greater than their rest energy, the neutrinos are moving at very nearly c and their energies are $E \approx pc$. (*a*) Show that the time difference in their arrival at Earth is given by

$$\Delta t = t_2 - t_1 = x\frac{u_1 - u_2}{u_1 u_2} \approx \frac{x\Delta u}{c^2}$$

where u_1 and u_2 are the respective speeds of the neutrinos and x is the distance traveled. (*b*) Show that when $E \gg mc^2$, the speed u is given by

$$\frac{u}{c} \approx 1 - \frac{1}{2}\left(\frac{mc^2}{E}\right)^2$$

(*c*) Using the result of (*b*) above, compute $u_1 - u_2$ for the energies and mass above and calculate Δt for $x = 1.7 \times 10^5\ c \cdot y$.

12-49. Show that the following decays conserve all lepton numbers.
(*a*) $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
(*b*) $\tau^- \rightarrow \mu^- + \nu_\mu + \nu_\tau$
(*c*) $n \rightarrow p + e^- + \bar{\nu}_e$
(*d*) $\tau^- \rightarrow \mu^- + \bar{\nu}_\mu$

12-50. A π^0 with energy 850 MeV decays in flight via the reaction $\pi^0 \rightarrow \gamma + \gamma$. Compute the angles made by the momenta of the gammas with the original direction of the π^0.

12-51. Test the following decays for violation of the conservation of energy, electric charge, baryon number, and lepton number:
(*a*) $\Lambda^0 \rightarrow p + \pi^-$
(*b*) $\Sigma^- \rightarrow n + p^-$
(*c*) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
Assume that linear and angular momentum are conserved. State which conservation laws (if any) are violated in each decay.

12-52. Consider the following decay chain:

$$\begin{aligned}
&\Omega^- \rightarrow \Xi^0 + \pi^- \\
&\Xi^0 \rightarrow \Sigma^+ + e^- + \bar{\nu}_e \\
&\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
&\Sigma^+ \rightarrow n + \pi^+ \\
&\pi^+ \rightarrow \mu^+ + \nu_\mu
\end{aligned}$$

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$
$$\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu$$

(a) Are all the final products shown stable? If not, finish the decay chain. (b) Write the overall decay reaction for Ω^- to the final products. (c) Check the overall decay reaction for the conservation of electric charge, baryon number, lepton number, and strangeness.

LEVEL III

12-53. The mass of the hydrogen atom is smaller than the sum of the masses of the proton and the electron, the difference being the binding energy. The mass of the π^+ is 139.6 MeV$/c^2$; however, the masses of the quarks of which it is composed are only a few MeV$/c^2$. How can that be explained?

12-54. (a) Calculate the total kinetic energy of the decay products for the decay $\Lambda^0 \rightarrow p + \pi^-$. Assume the Λ^0 is initially at rest. (b) Find the ratio of the kinetic energy of the pion to the kinetic energy of the proton. (c) Find the kinetic energies of the proton and the pion for this decay.

12-55. A Σ^0 particle at rest decays into a Λ^0 plus a photon. (a) What is the total energy of the decay products? (b) Assuming that the kinetic energy of the Λ^0 is negligible compared with the energy of the photon, calculate the approximate momentum of the photon. (c) Use your result for (b) to calculate the kinetic energy of the Λ^0. (d) Use your result for (c) to obtain a better estimate of the momentum and the energy of the photon.

12-56. In this problem, you will calculate the difference in the time of arrival of two neutrinos of different energy from a supernova that is 170,000 light-years away. Let the energies of the neutrinos be $E_1 = 20$ MeV and $E_2 = 5$ MeV, and assume that the mass of a neutrino is 2.4 eV$/c^2$. Because their total energy is so much greater than their rest energy, the neutrinos have speeds that are very nearly equal to c and energies that are approximately $E \approx pc$. (a) If t_1 and t_2 are the times it takes for neutrinos of speeds u_1 and u_2 to travel a distance x, show that

$$\Delta t = t_2 - t_1 = x\frac{u_1 - u_2}{u_1 u_2} \approx \frac{x\Delta u}{c^2}$$

(b) The speed of a neutrino of mass m_0 and total energy E can be found from Equation 2-10. Show that when $E \gg m_0c^2$, the speed u is given by

$$\frac{u}{c} \approx 1 - \frac{1}{2}\left(\frac{m_0c^2}{E}\right)^2$$

(c) Use the result for (b) to calculate $u_1 - u_2$ for the energies and mass given, and calculate Δt from the result (a) for $x = 170{,}000\ c \cdot$y. (d) Repeat the calculation in (c) using $m_0c^2 = 40$ eV for the rest energy of a neutrino.

12-57. There are three possible decay modes for the τ^-. (a) Draw the Feynman diagrams for each mode. (b) Which mode is the most probable? Explain why.

12-58. In a large accelerator, such as the Large Hadron Collider at CERN, the momentum of a proton in a circular orbit of radius R is given by $\mathbf{p} = 0.3\,R\mathbf{B}$ GeV$/c$, where $\mathbf{B}$ is the magnetic field. Derive this expression.

Problems

LEVEL I

Section 13-1 The Sun

13-1. Measurement of the Doppler shift of spectral lines in light from the east and west limbs of the Sun at the solar equator reveal that the tangential velocities of the limbs differ by 4 km/s. Use this result to compute the approximate period of the Sun's rotation ($R_\odot = 6.96 \times 10^5$ km).

13-2. The gravitational potential energy U of a self-gravitating spherical body of mass M and radius R is a function of the details of the mass distribution. For the Sun, $U_\odot = -2GM_\odot^2 R_\odot$. What would be the approximate lifetime of the Sun, radiating at its present rate, if the source of its emitted energy were entirely derived from gravitational contraction? ($M_\odot = 1.99 \times 10^{30}$ kg.)

Section 13-2 The Stars

13-3. Lithium, beryllium, and boron ($Z = 3$, 4, and 5, respectively) have very low abundances in the cosmos compared to many heavier elements (see Figure 13-33). Considering the fusion of He to C, explain these low abundances.

13-4. The Sun is moving with speed 2.5×10^5 m/s in a circular orbit about the center of the Galaxy. How long (in Earth years) does it take to complete one orbit? How many orbits has it completed since it was formed?

13-5. The reason that massive neutrinos were considered as a candidate for solving the missing mass problem is that, at the conclusion of the lepton era, the universe contained about equal numbers of photons and neutrinos. They are still here, for the most part. The former can be observed and their density is measured to be about 500 photons/cm^3; thus, there must be about that number density of neutrinos in the universe, too. If neutrinos have a nonzero mass and if the cosmological expansion has reduced their average speed so that their energy is now primarily mass, what would be the individual neutrino mass (in eV/c^2) necessary to account for the missing mass of the universe? Recall that the observed mass of the stars and galaxies (including the dust and gas) accounts for only about 4 percent of that needed to close the universe.

13-6. Using data from Table 13-3, construct a graph that demonstrates the validity of Equation 13-17.

13-7. Recalling that the light-year $c \cdot$ y is the distance light travels in one year, compute in meters the distance equivalent to 1 light-second, 1 light-minute, 1 light-hour, and 1 light-day.

Section 13-3 The Evolution of Stars

13-8. A unit of length often used by astronomers to measure distances in "nearby" space is the parsec (pc), defined as the distance at which a star subtends a parallax angle of one arc second due to Earth's orbit around the Sun (see Equation 13-11 and Example 13-4). The practical limit of such measurements is 0.01 arc second. (*a*) How many light-years is 1 pc? (*b*) If the density of stars in the Sun's region of the Milky Way is 0.08 star/pc^3, how many stars could, in principle, have their distances from us measured by the trigonometric parallax method?

13-9. Astronomers often use the *apparent magnitude m* as a means of comparing the visual brightness of stars and relating the comparison to the luminosity and distance to "standard" stars, such as the Sun (see Equation 13-9). The difference in the apparent magnitudes of two stars m_1 and m_2 is defined as $m_2 - m_1 = 2.5 \log (f_1/f_2)$, a relation based on the logarithmic response of the human eye to the brightness of objects. Pollux, one of the "twins" in the constellation Gemini, has apparent magnitude 1.16 and is 12 pc away.

Betelgeuse, the star at Orion's right shoulder, has apparent magnitude 0.41. How far away is Betelgeuse if they have the same luminosity?

13-10. Using the H-R diagram (Figure 13-17), determine the effective temperature and the luminosity of a star whose mass is (*a*) 0.3 $M_\odot$ and (*b*) 3 $M_\odot$. (*c*) Compute the radius of each star. (*d*) Determine their expected lifetimes relative to that of the Sun.

13-11. Two stars in a binary system are 100 $c \cdot y$ from Earth and separated from each other by 10^8 km. What is the angular separation of the stars in arc seconds? In degrees?

Section 13-4 Cataclysmic Events

13-12. Compute the energy required (in MeV) to produce each of the photodisintegration reactions in Equations 13-18 and 13-19.

13-13. The gas shell of the planetary nebula shown in Figure 13-18 is expanding at 24 km/s. Its diameter is 1.5 $c \cdot y$. (*a*) How old is the gas shell? (*b*) If the central star of the planetary nebula is 12 times as luminous as the Sun and 15 times hotter, what is the radius of the central star in units of $R_\odot$?

Section 13-5 Final States of Stars

13-14. Calculate the Schwarzschild radius of a star whose mass is equal to that of (*a*) the Sun, (*b*) Jupiter, (*c*) Earth. (The mass of Jupiter is approximately 318 times that of Earth.)

13-15. Consider a neutron star whose mass equals 2 $M_\odot$. (*a*) Compute the star's radius. (*b*) If the neutron star is rotating at 0.5 rev/s and assuming its density to be uniform, what is its rotational kinetic energy? (*c*) If its rotation slows by 1 part in 10^8 per day and the lost kinetic energy is all radiated, what is the star's luminosity?

13-16. If the 90 percent of the Milky Way's mass that is "missing" resides entirely in a large black hole at the center of the Galaxy, what would be the black hole's (*a*) mass and (*b*) radius?

Section 13-6 Galaxies

13-17. Redshift measurements for a particular galaxy indicate that it has a recession velocity of 72,000 km/s. (*a*) Compute the distance to the galaxy. (*b*) The value of Hubble's constant depends critically on calibration distance measurements, which are difficult to make. If the calibration distance measurements are in error by 10 percent, by how much is the age calculated from Equation 13-28 in error?

13-18. The bright core of a certain Seyfert galaxy had a luminosity of 10^{10} $L_\odot$. The luminosity increased by 100 percent in a period of 18 months. Show that this means that the energy source of the core is less than 9.45×10^4 AU in diameter. How does this compare to the diameter of the Milky Way?

13-19. The wavelength of the Hα line in the hydrogen spectrum is 656.3 nm. Use Hubble's law to determine the wavelength of the Hα line emitted from galaxies at distances of (*a*) 5×10^6 $c \cdot y$, (*b*) 50×10^6 $c \cdot y$, (*c*) 500×10^6 $c \cdot y$, and (*d*) 5×10^9 $c \cdot y$ from Earth.

Section 13-7 Cosmology and Gravitation

13-20. Evaluate Equation 13-33 for the critical density of the universe.

Section 13-8 Cosmology and the Evolution of the Universe

13-21. Cosmological theory suggests that the average separation of galaxies, that is, the scale of the universe, is inversely proportional to the absolute temperature. If that is true, relative to the present size, how large was the universe compared to the scale today (*a*) 2000 years ago, (*b*) 10^6 years ago, (*c*) $t = 10$ s after the Big Bang, (*d*) when $t = 1$ s, and (*e*) when $t = 10^{-6}$ s?

13-22. Determine the value of the mass density of the universe for $t =$ Planck time. How does this compare to the density of the proton? Of osmium?
13-23. At what wavelength is the blackbody radiation distribution of the cosmic microwave background at a maximum?
13-24. How long after the Big Bang did it take the universe to cool to the threshold temperature for the formation of muons? What would be the mass of a particle-antiparticle pair that could be formed by the average energy of the current 2.725 K background radiation?
13-25. Show that the present mass density of the universe $\rho_0 = R(t)\rho(t)$.

LEVEL II

13-26. If Hubble's law is true for an observer in the Milky Way (i.e., us), prove that it must also be true for observers in other galaxies. (*Hint:* Use the vector property of the velocity.)
13-27. Find the minimum magnitude of the radius a that a dust particle in orbit around the Sun may have in order to avoid being blown out of the solar system by the Sun's radiation pressure. Assume that the particle is a sphere of mass m with the same density ρ as Earth, $5500\ \text{kg/m}^3$. Ignore the solar wind and the solar magnetic field.
13-28. Show that the mass density of the universe at redshift z is given by $\rho(z) = \rho(1 + z)^3$.
13-29. When the Sun was formed, about 75 percent of its mass was hydrogen, of which only about 13 percent ever becomes available for fusion. (The rest is in regions of the Sun where the temperature is too low for fusion reactions to occur.) $M_\odot = 2 \times 10^{30}$ kg and the Sun fuses about 6×10^{11} kg/s. (*a*) Compute the total mass of hydrogen available for fusion during the Sun's lifetime. (*b*) How long (in years) will the Sun's initial supply of hydrogen last? (*c*) Since the solar system is currently about 4.6×10^9 y old, when should we begin to worry about the Sun running out of hydrogen for fusion?
13-30. Supernova SN1987A was first visible at Earth in 1987. (*a*) How many years B.P. (before present) did the explosion occur? (*b*) If protons with 100 GeV of kinetic energy were produced in the event, when should they arrive at Earth?
13-31. Assume that the Sun when it first formed was composed of 70 percent hydrogen. How many hydrogen nuclei were there in the Sun at that time? How much energy would ultimately be released if all of the hydrogen nuclei fused into helium? Astrophysicists have predicted that the Sun can radiate energy at its current rate until about 23 percent of the hydrogen has been "burned." What total lifetime for the Sun does that prediction imply? Compare these results with the corresponding ones from Problem 13-29.
13-32. Kepler's third law states that the square of a planet's orbital speed is proportional to the cube of its average orbital radius. Use Kepler's third law to answer each of the following questions. (*a*) The Moon's orbital radius is 3.84×10^5 km and it orbits Earth once every 27.3 d. Neglecting the moon's mass, compute the mass of Earth. (*b*) Io (one of Jupiter's moons) orbits Jupiter once every 42.5 h in a near-circular orbit of average radius 4.22×10^5 km. Neglecting Io's mass, compute the mass of Jupiter. (*c*) Compute the orbital period of the International Space Station as it orbits 300 km above Earth's surface. (*d*) Charon, a moon of Pluto, orbits that body once every 6.4 d at an average distance of 1.97×10^4 km. Compute the total mass of Pluto and Charon. What fraction of Earth's mass is this? (*e*) Using the data for the star S2, compute the volume (upper limit) that confines the black hole at the center of the Milky Way. Compare the result with the volume of the Sun.
13-33. Consider an eclipsing binary whose orbital plane is parallel to our line of sight. Doppler measurements of the radial velocity of each component of the binary are shown in Figure 13-36. Assume that the mass $m_1 > m_2$ and that the orbits of each component about the center of mass are circular. (*a*) What is the period T and the angular frequency ω of the binary? (*b*) Show that in this case $(m_1 + m_2) = (\omega^2 r^3)/G$, where $r =$ separation of the binary. (*c*) Compute the values of m_1, m_2, and r from the data in the v versus t graph.

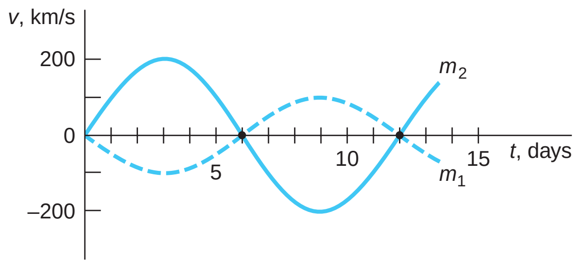

FIGURE 13-36

13-34. Prove that the total energy of Earth's orbital motion $E = (mv^2/2) + (-GM_\odot m/r)$ is equal to one-half of its gravitational potential energy $(-GM_\odot m/r)$, where r is Earth's orbit radius.

13-35. Given the currently accepted value of the Hubble constant and the fact that the average matter density of the universe is one H atom/m^3, what creation rate of new H atoms would be necessary in a steady-state model to maintain the present mass density, even though the universe is expanding? (Give your answer in H atoms/m^3 per 10^6 years.) Would you expect such a spontaneous creation rate to be readily observable?

Level III

13-36. The ability of a planet to retain particular gases in an atmosphere depends on the temperature that its atmosphere has (or would have) and the escape velocity for the planet. In general, if the average speed of a particular gas molecule exceeds 1/6 of the escape velocity, that gas will disappear from the atmosphere in about 10^8 years. (*a*) Graph the average speed of H_2O, CO_2, O_2, CH_4, H_2, and He from 50 K to 1000 K. On the same graph show the points representing 1/6 of the escape velocity versus average temperature of the atmosphere for the planets in Table 13-5 below. (*b*) Show that the escape speed v from a planet is given by

$$\frac{v}{v_{\text{Earth}}} = \sqrt{\frac{(M/M_{\text{Earth}})}{(R/R_{\text{Earth}})}}$$

(*c*) Which of the six gases plotted probably would and would not currently be found in the atmospheres of the solar system bodies in the table? Explain *each* answer briefly.

Table 13-5 Atmospheric temperatures

Average T_{atm} (K)	Planet	M/M_{Earth}	R/R_{Earth}
300	Earth	1.00	1.00
390	Venus	0.81	0.95
600	Mercury	0.06	0.38
150	Jupiter	318.00	11.00
60	Neptune	17.00	3.90
290	Mars	0.11	0.53

13-37. Using the parallax technique, compute the distance to (*a*) Alpha Centauri (parallax angle 0.742 arc second) and (*b*) Procyon (parallax angle 0.0286 arc second). Express each answer in both light-years and parsecs.

13-38. As the Sun evolves into a red giant star, suppose that its luminosity increases by a factor of 10^2. Show that Earth's oceans will evaporate, but that the water vapor will not escape from the atmosphere.

13-39. The approximate mass of dust in the Galaxy can be computed from the observed extinction of starlight. Assuming the mean radius of dust grains to be R with a uniform number density n grains/cm^3, (*a*) show that the mean free path d_0 of a photon in interstellar dust is given by $d_0 = 1/(n\pi R^2)$. (*b*) Starlight traveling toward an Earth observer a distance d from the star has intensity

$$I = I_0 e^{-d/d_0}$$

In the vicinity of the Sun a measurement of I yields $d_0 = 3000\ c \cdot \text{y}$. If $R = 10^{-5}$ cm, calculate n. (*c*) The average mass density of solid material in the Galaxy is 2 g/cm^3 and in the disk the density of stars is about $1\ M_\odot/300\ (c \cdot \text{y})^3$. Compute the ratio of the mass density of dust to the mass density of stars, assuming $1\ M_\odot$ in $300\ (c \cdot \text{y})^3$.

13-40. The supernova SN1987A certainly produced some heavy elements. Compared to the energy released in fusing 56 ^{1}H atoms into one ^{56}Fe atom starting from the proton-proton cycle, how much energy would be required to fuse two ^{56}Fe atoms into one ^{112}Cd atom?

13-41. Current theory suggests that black holes evaporate by the emission of Hawking radiation in a time t that depends on the mass M of the black hole according to the following relation:

$$t = (1.024 \times 10^4 \pi^2\ \text{m}^3/\text{s}^2) G^2 M^2 / hc^4$$

(*a*) Explain without calculating anything why the formula implies that high-mass black holes have longer lifetimes than low-mass ones and why the rate of evaporation accelerates as the black hole loses mass. (*b*) Compute the lifetime of a black hole whose mass equals $1\ M_\odot$. Compare this time with the current age of the universe. (*c*) According to some theories, the largest black hole that could conceivably form would have a mass $10^{12}\ M_\odot$, of the order of the mass of an entire galaxy. What would be the lifetime of a black hole that large?