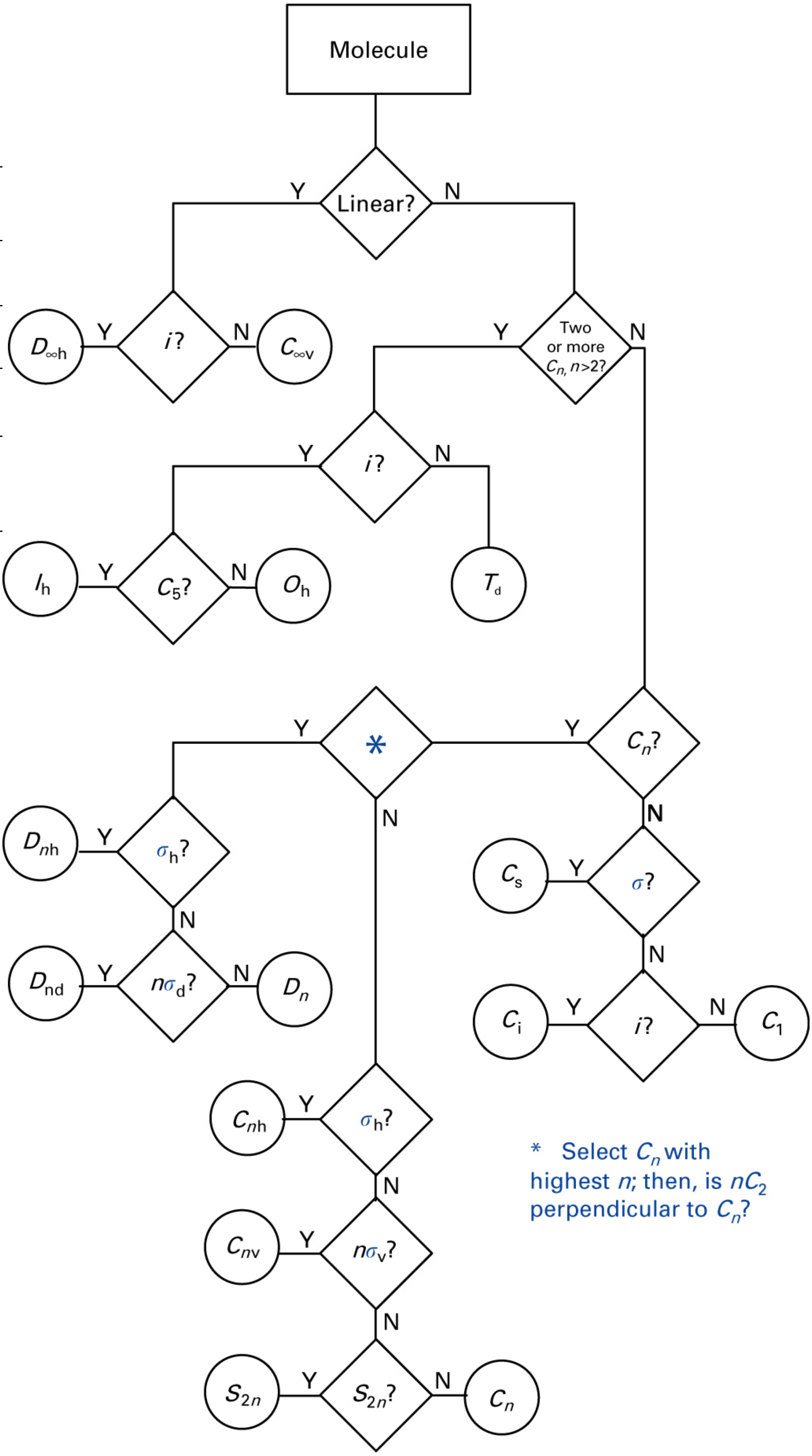
1. The Groups C_1, C_s, C_i

C_1 (1)	E
A	1

$C_s = C_h$ (m)	E	σ_h
A′	1	1
A″	1	–1

$C_i = S_2$ ($\bar{1}$)	E	i
A_g	1	1
A_u	1	–1



4. The Groups C_{nv} (n = 2, 3, 4, 5, 6)

C_{2v} (2*mm*)	E	C_2	$\sigma_v(xz)$	$\sigma'_v(yz)$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	–1	–1	R_z	xy
B_1	1	–1	1	–1	x, R_y	xz
B_2	1	–1	–1	1	y, R_x	yz

C_{3v} (3*m*)	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	–1	R_z	
E	2	–1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(xz, yz)$

C_{4v} (4*mm*)	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1		xy
E	2	0	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

C_{5v}	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	R_z	
E_1	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

C_{6v} (6*mm*)	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	–1		
B_2	1	–1	1	–1	–1	1		
E_1	2	1	–1	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$

5. The Groups C_{nh} (n = 2, 3, 4, 5, 6)

C_{2h} (2/m)	E	C_2	I	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	–1	1	–1	R_x, R_y	xz, yz
A_u	1	1	–1	–1	z	
B_u	1	–1	–1	1	x, y	

C_{3h} ($\bar{6}$)	E	C_3	C_3^2	σ_h	S_3	S_3^5		$\varepsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E'	1	ε	ε^*	1	ε	ε^*	(x, y)	$(x^2 - y^2, 2xy)$
	1	ε^*	ε	1	ε^*	ε		
A''	1	1	1	–1	–1	–1	z	
E''	1	ε	ε^*	–1	$-\varepsilon$	$-\varepsilon^*$	(R_x, R_y)	(xz, yz)
	1	ε^*	ε	–1	$-\varepsilon^*$	$-\varepsilon$		

C_{4h} (4/m)	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
B_g	1	–1	1	–1	1	–1	1	–1		$(x^2 - y^2, 2xy)$
E_g	1	i	–1	–i	1	i	–1	–i	(R_x, R_y)	(xz, yz)
	1	–i	–1	i	1	–i	–1	i		
A_u	1	1	1	1	–1	–1	–1	–1	z	
B_u	1	–1	1	–1	–1	1	–1	1		
E_u	1	i	–1	–i	–1	–i	1	i	(x, y)	
	1	–i	–1	i	–1	i	1	–i		

6. The Groups D_{nh} (n = 2, 3, 4, 5, 6)

D_{2h} (mmm)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	–1	–1	1	1	–1	–1	R_z	xy
B_{2g}	1	–1	1	–1	1	–1	1	–1	R_y	xz
B_{3g}	1	–1	–1	1	1	–1	–1	1	R_x	yz
A_u	1	1	1	1	–1	–1	–1	–1		
B_{1u}	1	1	–1	–1	–1	–1	1	1	z	
B_{2u}	1	–1	1	–1	–1	1	–1	1	y	
B_{3u}	1	–1	–1	1	–1	1	1	–1	x	

D_{3h} ($\bar{6}m2$)	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		x^2+y^2, z^2
A_2'	1	1	–1	1	1	–1	R_z	
E'	2	–1	0	2	–1	0	(x, y)	$(x^2-y^2, 2xy)$
A_1''	1	1	1	–1	–1	–1		
A_2''	1	1	–1	–1	–1	1	z	
E''	2	–1	0	–2	1	0	(R_x, R_y)	(xy, yz)

D_{4h} ($4/mmm$)	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	–1	–1	1	1	1	–1	–1	R_z	
B_{1g}	1	–1	1	1	–1	1	–1	1	1	–1		x^2-y^2
B_{2g}	1	–1	1	–1	1	1	–1	1	–1	1		xy
E_g	2	0	–2	0	0	2	0	–2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	–1	–1	–1	–1	–1		
A_{2u}	1	1	1	–1	–1	–1	–1	–1	1	1	z	
B_{1u}	1	–1	1	1	–1	–1	1	–1	–1	1		
B_{2u}	1	–1	1	–1	1	–1	1	–1	1	–1		
E_u	2	0	–2	0	0	–2	0	2	0	0	(x, y)	

7. The Groups D_{nd} (n = 2, 3, 4, 5, 6)

$D_{2d} = V_d$ $(\overline{42})m$	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1	z	xy
E	2	0	–2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_{3d} $(\overline{3})m$	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	–1	1	1	–1	R_z	
E_g	2	–1	0	2	–1	0	(R_x, R_y)	$(x^2 - y^2, 2xy)$ (xz, yz)
A_{1u}	1	1	1	–1	–1	–1		
A_{2u}	1	1	–1	–1	–1	1	z	
E_u	2	–1	0	–2	1	0	(x, y)	

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	1	–1		
B_2	1	–1	1	–1	1	–1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	–2	0	0	(x, y)	
E_2	2	0	–2	0	2	0	0		$(x^2 - y^2, 2xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	–2	0	0	(R_x, R_y)	(xz, yz)

9. The Cubic Groups

T (23)	E	$4C_3$	$4C_3^2$	$3C_2$		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	1		$x^2 + y^2 + z^2$
E	$\begin{cases} 1 \\ 1 \end{cases}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$		$(\sqrt{3}\,(x^2 - y^2) 2z^2 - x^2 - y^2)$
T	3	0	0	–1	(x, y, z) (R_x, R_y, R_z)	(xy, xz, yz)

T_d ($\bar{4}3m$)	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_2	1	1	1	–1	–1		
E	2	–1	2	0	0		$(2z^2 - x^2 - y^2,\ \sqrt{3}\,(x^2 - y^2)$
T_1	3	0	–1	1	–1	(R_x, R_y, R_z)	
T_2	3	0	–1	–1	1	(x, y, z)	(xy, xz, yz)

T_h ($m3$)	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^2$	$3\sigma_d$		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
E_g	$\begin{cases} 1 \\ 1 \end{cases}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\,(x^2 - y^2))$
T_g	3	0	0	–1	3	0	0	–1	(R_x, R_y, R_z)	(xy, yz, xz)
A_u	1	1	1	1	–1	–1	–1	–1		
E_u	$\begin{cases} 1 \\ 1 \end{cases}$	$\begin{matrix} \varepsilon \\ \varepsilon^* \end{matrix}$	$\begin{matrix} \varepsilon^* \\ \varepsilon \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$	$\begin{matrix} -\varepsilon \\ -\varepsilon^* \end{matrix}$	$\begin{matrix} -\varepsilon^* \\ -\varepsilon \end{matrix}$	$\begin{matrix} -1 \\ -1 \end{matrix}$		
T_u	3	0	0	–1	–3	0	0	1	(x, y, z)	

O (432)	E	$8C_3$	$3C_2$	$6C_4$	$6C_2'$		
A_1	1	1	1	1	1		$x^2 + y^2 + z^2$
A_2	1	1	1	–1	–1		
E	2	–1	2	0	0		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\,(x^2 - y^2))$
T_1	3	0	–1	1	–1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	0	–1	–1	1		(xy, xz, yz)

11. The Groups $C_{\infty v}$ and $D_{\infty h}$

$C_{\infty v}$	E	C_2	$2C_\infty^\phi$	…	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	1	…	1	z	x^2+y^2, z^2
$A_2 \equiv \Sigma^-$	1	1	1	…	−1	R_z	
$E_1 \equiv \Pi$	2	−2	$2\cos\phi$	…	0	(x,y) (R_x,R_y)	(xz, yz)
$E_2 \equiv \Delta$	2	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
$E_3 \equiv \Phi$	2	−2	$2\cos 3\phi$	…	0		
…	…	…	…	…	…		
…	…	…	…	…	…		

$D_{\infty h}$	E	$2C_\infty^\phi$	…	$\infty\sigma_v$	i	$2S_\infty^\phi$	…	∞C_2		
Σ_g^+	1	1	…	1	1	1	…	1		x^2+y^2, z^2
Σ_g^-	1	1	…	−1	1	1	…	−1	R_z	
Π_g	2	$2\cos\phi$	…	0	2	$-2\cos\phi$	…	0	(R_x, R_y)	(xz, yz)
Δ_g	2	$2\cos 2\phi$	…	0	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
…	…	…	…	…	…	…	…	…		
Σ_u^+	1	1	…	1	−1	−1	…	−1	z	
Σ_u^-	1	1	…	−1	−1	−1	…	1		
Π_u	2	$2\cos\phi$	…	0	−2	$2\cos\phi$	…	0	(x,y)	
Δ_u	2	$2\cos 2\phi$	…	0	−2	$-2\cos 2\phi$	…	0		
…	…	…	…	…	…	…	…	…		

Direct Products

1. General rules

(*a*) For point groups in the lists below that have representations *A, B, E, T* without subscripts, read $A_1 = A_2 = A$, etc.

(*b*)

	g	u
g	g	u
u		g

	′	″
′	′	″
″		′

(*c*) Square brackets [ ] are used to indicate the representation spanned by the antisymmetrized product of a degenerate representation with itself.

Examples

For D_{3h} $E' \times E''$ $A_1'' + A_2'' + E$

For D_{6h} $E_{1g} \times E_{2g} = 2B_g + E_{1g}$.

2. For C_2, C_3, C_6, D_3, D_6, C_{2v}, C_{3v}, C_{6v}, C_{2h}, C_{3h}, C_{6h}, D_{3h}, D_{6h}, D_{3d}, S_6

	A_1	A_2	B_1	B_2	E_1	E_2
A_1	A_1	A_2	B_1	B_2	E_1	E_2
A_2		A_1	B_2	B_1	E_1	E_2
B_1			A_1	A_2	E_2	E_1
B_2				A_1	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$B_1 + B_2 + E_1$
E_2						$A_1 + [A_2] + E_2$

3. For D_2 , D_{2h}

	A	B_1	B_2	B_3
A	A	B_1	B_2	B_3
B_1		A	B_3	B_2
B_2			A	B_1
B_3				A